Scheduling Live Interactive Narratives with Mixed-Integer Linear Programming

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Outline

- Live interactive narratives
- Describe our problem formulation
- Live Interactive Narrative Scheduler
- Evaluate scalability and feasibility
Live Interactive Narratives (LINs)

multiple players participate in a predesigned narrative taking on fictional roles and interacting with real world objects and actors
Bad News

45 min
narrative arc
9 mil. players
Shared actors, props, locations
Challenges with Live Interactive Narratives

- High cost for authoring and reauthoring
- Time taken to complete events and interactions is inconsistent
- Authoring for consistent story beats or dramatic tension
- Mitigation of real world operating costs
People don’t understand time. People assume that time is a strict progression of cause to effect, but actually from a non-linear, non-subjective viewpoint, it's more like a big ball of wibbly-wobbly, timey-wimey stuff.
— Doctor Who (Series 3, Episode 10 - Blink)
Temporal Challenges with Narratives

- Dependency graph with temporal durations of events.
Temporal Challenges with Narratives

- Dependency graph with temporal durations of events.
- Contingent (or Uncontrollable) events

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- Requirement (or Controllable) events
Temporal Challenges with Narratives

- Dependency graph with temporal durations of events.
- Contingent (or Uncontrollable) events
- Requirement (or Controllable) events
- Resource availability
  - Original available
  - Produced during an event
  - Consumed during an event
Temporal Challenges with Narratives

- Dependency graph with temporal durations of events.
- Contingent (or Uncontrollable) events
- Requirement (or Controllable) events
- Resource (original, produced, consumed)
- Plot choices!
Problem Statement

(pointer text)

- **Given:**
  - Uncontrollable durations
  - Constraints on controllable durations
  - Constraints on resources
  - Plot choices
- **No peeking into the future**
- **Change the duration**
  - Controllable time points
  - Any uncontrollable observed duration
Live Interactive Narratives
Scheduling Problem (LINSP)
Live Interactive Narrative Scheduling Problem (LINSP)

- Formulated as a mixed integer linear programming constraint satisfaction problem
- A LINSP problem is defined as a tuple

\[ \langle V, E_{req}, E_{ctg}, U, L, A, b, R, M \rangle \]

- **Temporal / Narrative Bounds**
- **Resource Constraints**
- **Mutually Exclusive Constraints**
Temporal Constraints

Shortest Path Constraints
\[ l_{AC} \leq u_{AB} + l_{BC} \leq u_{AC} \]
\[ l_{AC} \leq l_{AB} + u_{BC} \leq u_{AC} \]
\[ u_{AC} \leq u_{AB} + u_{BC} \]
\[ l_{AB} + l_{BC} \leq l_{AC} \]

Precede Constraints
\[ u_{AB} \leq l_{AC} - l_{BC} \]
\[ l_{AB} \geq u_{AC} - u_{BC} \]

Unordered Constraints
\[ (l_{BC} < 0) \lor \left( u_{AB} \leq l_{AC} - l_{BC} \right) \]
\[ l_{AB} \geq u_{AC} - u_{BC} \]

Wait Constraints for this triangle
\[ w_{ABC} \geq u_{AC} - u_{BC} \]

Wait Constraints for overall story
\[ l_{AB} \geq \min(l_{AX}, w_{ABX}) \]
\[ w_{ADX} \geq w_{ABX} - l_{DB} \]
\[ (w_{ABX} < 0) \lor (w_{ADX} \geq w_{ABX} - l_{DB}) \]

Sequencing for resources used
\[ l_{S_i S_j} \geq u_{S_i E_i} - M \times (1 - x_{i,j}), \quad i < j \]
\[ x_{i,j} + x_{j,i} \leq 1, \quad \forall (i, j) \in \{0, \ldots, n + 1\}^2 \]
\[ x_{i,k} \geq x_{i,j} + x_{j,k} - 1, \quad \forall (i, j, k) \in \{0, \ldots, n + 1\}^3 \]
Live Interactive Narrative Temporal Constraints

Prior Work

- Shortest Path Constraints
- Follow Constraints
  If activity A follows B
- Precede Constraints
  If activity A occurs before or at the same time as B
- Unordered Constraints
  If activity A and B are scheduled independently
- Wait Constraints
  Activity B must wait for either
  1) a minimum time, or 2) till after a specific event A is over

Our Contribution

- Plot Choice Constraints
  Accounts for interactive narrative choices that may be mutually exclusive to each other
- Resource Flow Constraints
  Accounts for available resources flowing in a narrative
- Resource Consumption Constraints
  Accounts for activities that consume resources
- Resource Production Constraints
  Accounts for activities that can produce or consume resources
The Great Dragon Adventure

1. Merlin [10,90]
   - Sword [30,70]
   - Fight bandit [20,50]
   - Find treasure [20,70]

2. Magic [20,100]
   - Sword [30,70]
   - Fight bandit [20,50]
   - Find treasure [20,70]


4. Get spell book [50,120]

5. Robbery [5,80]
   - Magic [20,100]

6. Be imprisoned [30,100]
   - Get spell book [50,120]
   - Companions arrive [0,35]

7. Rob merchant [10,40]

8. Be imprisoned [30,100]
   - Companions arrive [0,35]
   - Group battle [5,15]

9. Be imprisoned [30,100]
   - Companions arrive [0,35]
   - Group battle [5,15]

10. Companions arrive [0,35]

11. Group battle [5,15]

Final dragon battle [30,40]

[120,180]
Graph showing the sequence of events:

- **Start Battle**
- **Battle with sword**
- **Companion Arrives**
- **End Battle**

Transitions:
- From 9 to 11: [30, 40] (Requirement (controllable))
- From 9 to 10: [0, 35] (Contingent (uncontrollable))
- From 10 to 11: [5, 15] (Contingent (uncontrollable))
- From 10 to 9: [0, 35] (Contingent (uncontrollable))

Events:
- Battle
- Companion Arrives
Start Battle

9

Battle with sword

[30, 40]

End Battle

11

Companion Arrives

10

= 0.35

> 15 Fails!

Requirement (controllable)

Contingent (uncontrollable)
Battle with sword

Start Battle

End Battle

Companion Arrives

Requirement (controllable)

Contingent (uncontrollable)

= 30

[30, 40]

= 35?

[0, 35]

< 5 Fails!

[5, 15]

[0, 35]
Dynamically Controllable!

- Requirement (controllable)
- Contingent (uncontrollable)
Simple Temporal Problem with Uncertainty
Simple Temporal Problem with Uncertainty

Graph: 1. Merlin
   2. Sword, Magic
   3. Fight bandit
   4. Find treasure
   5. Magic Duel
   6. Get spell book
   7. Robbery
   8. Rob merchant
   9. Companions arrive
  10. Group battle
  11. Final dragon battle

Nodes: 120, 180, 10, 90, 30, 70, 20, 5, 40, 10, 30, 10, 40, 20, 60, 50, 110, 30, 100, 25, 25, 30, 40, 5, 15
Entire Ctg $[10,90]$ interval is covered!

**LIN Scheduler Plot Choices Constraint**

We can allow individual plot choices and routes to not be feasible during a subset or interval of time of the original bound, as long as together they collectively cover the entire original interval.
Mutually Exclusive Plot Choices Constraints

\[ \text{Ctg}_{12} = [10,90] \text{ Total time taken by player for training} \]

\begin{itemize}
  \item Sword route feasible for [15,80]
  \item Magic route feasible for [10,70]
  \item Robbery route feasible for [50,90]
\end{itemize}

- Every feasible plot choice interval is a node
Mutually Exclusive Plot Choices Constraints

\[ \text{Ctg}_{12} = [10,90] \text{ Total time taken by player for training} \]

- Sword route feasible for [15,80]
- Magic route feasible for [10,70]
- Robbery route feasible for [50,90]

Every feasible plot choice interval is a node
Mutually Exclusive Plot Choices Constraints

\[ C_{tg12} = [10,90] \text{ Total time taken by player for training} \]

- Sword route feasible for \([15,80]\)
- Magic route feasible for \([10,70]\)
- Robbery route feasible for \([50,90]\)

- Every feasible plot choice interval is a node
- Child Edge \(\rightarrow\) Overlap in temporal interval
Mutually Exclusive Plot Choices Constraints

Every feasible plot choice interval is a node
Child Edge —> Overlap in temporal interval
Parent Outward Edge —> Lower bound covered
Mutually Exclusive Plot Choices Constraints

\[ C_{t_1, t_2} = [10, 90] \text{ Total time taken by player for training} \]

- Sword route feasible for \([15, 80]\)
- Magic route feasible for \([10, 70]\)
- Robbery route feasible for \([50, 90]\)

- Every feasible plot choice interval is a node
- Child Edge \(\rightarrow\) Overlap in temporal interval
- Parent Outward Edge \(\rightarrow\) Lower bound covered
- Parent Inward Edge \(\rightarrow\) Upper bound covered
Mutually Exclusive Plot Choices Constraints

- Every feasible plot choice interval is a node
- Child Edge —> Overlap in temporal interval
- Parent Outward Edge —> Lower bound covered
- Parent Inward Edge —> Upper bound covered
- Cycle found must include parent and at least one mutex node

\[ C_{tg_{12}} = [10,90] \] Total time taken by player for training

Sword route feasible for \([15,80]\)

Magic route feasible for \([10,70]\)

Robbery route feasible for \([50,90]\)
Every feasible plot choice interval is a node
Child Edge —> Overlap in temporal interval
Parent Outward Edge —> Lower bound covered
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Cycle found must include parent and at least one mutex node
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Mutually Exclusive Plot Choices Constraints

Every feasible plot choice interval is a node.
Child Edge $\rightarrow$ Overlap in temporal interval
Parent Outward Edge $\rightarrow$ Lower bound covered
Parent Inward Edge $\rightarrow$ Upper bound covered
Cycle found must include parent and at least one mutex node

- Sword route feasible for $[15,80]$
- Magic route feasible for $[10,70]$
- Robbery route feasible for $[50,90]$

Cycle found $\rightarrow$ $\tau_0$ to $\tau_1$ to $\tau_2$ to $\tau_3$
Mutually Exclusive Plot Choices Constraints

- Every feasible plot choice interval is a node
- Child Edge $\rightarrow$ Overlap in temporal interval
- Parent Outward Edge $\rightarrow$ Lower bound covered
- Parent Inward Edge $\rightarrow$ Upper bound covered
- Cycle found must include parent and at least one mutex node

$\text{Ctg}_{12} = [10,90]$ Total time taken by player for training

Sword route feasible for $[15,80]$  
Magic route feasible for $[10,70]$  
Robbery route feasible for $[50,90]$
Every feasible plot choice interval is a node.
- Child Edge —> Overlap in temporal interval
- Parent Outward Edge —> Lower bound covered
- Parent Inward Edge —> Upper bound covered
- Cycle found must include parent and at least one mutex node

\[ C_{12} = [10,90] \] Total time taken by player for training

Sword route feasible for [15,80]
Magic route feasible for [10,70]
Robbery route feasible for [50,90]
1. Merlin
   [10, 90]

2. Sword
   [30, 45]

3. Fight bandit
   [20, 50]

4. Find treasure
   [20, 40]

5. Magic Duel
   [20, 35]

6. Get spell book
   [50, 80]

7. Rob Merchant
   [10, 40]

8. Be Imprisoned
   [30, 70]

9. Companions arrive
   [25, 25]

10. Final dragon battle
    [30, 40]

11. Group Battle
    [5, 15]
Evaluation

- NP hard!
- Lack of datasets
- Larger temporal durations imply the solution space explodes
- Dragon Adventure:
  - Number of linear programming variables: 3148
  - Total evaluated combinations (solution space): 1,939,168
  - CPU Solver Time: 20.3 ms
Evaluation: Test Set I

- 77 Plot Graphs
- 50% Contingent Events
- 2 to 5 Narratives running simultaneously
- 0-3 Resources per event
- 385 Total problems

Solver:
Gurobi Solver
2.7 GHz Intel Xeon E5 12-Core
64GB RAM
Evaluation: Test Set II

119 Random Plot Graphs
Varied % Contingent Events

Solver:
Gurobi Solver
2.7 GHz Intel Xeon E5 12-Core
64GB RAM
Evaluation: Test Set III

- Narrative Optimisations Tested:
  - LINSP Feasibility Only
  - Maximise Player Choices
  - Maximise Flexibility for Authoring
  - Maximise Rigidity of Requirement Events
  - Reducing Wait Time
LINSP Evaluation: Test Set III
Discussion

More Flexible
Allows more improvisation

More Rigid
Better PCG, more control
Conclusion

- Identify temporal challenges
- Formulation - temporal uncertainty, resources, plot choices
- Evaluated scalability and feasibility
- Evaluated several optimisation functions
Future Work

- Dataset of narratives bound with temporal durations
- Real time execution
- Mediation
Live Interactive Narrative Scheduling Problem (LINSP) allows for a single designed narrative game to be scaled to account for temporal uncertainty, resources, multiple players and multiple simultaneous instances easily!

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Thank You